Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 23, 2003*
Accepted: *May 5, 2003*

Of bounces, branes and bounds

Allan Joseph Michael Medved

Department of Physics and Theoretical Physics Institute, University of Alberta
Edmonton, Canada T6G-2J1
E-Mail: amedved@phys.ualberta.ca

Abstract: Some recent studies have considered a Randall-Sundrum-like brane world evolving in the background of an anti-de Sitter Reissner-Nordstrom black hole. For this scenario, it has been shown that, when the bulk charge is non-vanishing, a singularity-free "bounce" universe will always be obtained. However, for the physically relevant case of a de Sitter brane world, we have recently argued that, from a holographic (c-theorem) perspective, such brane worlds may not be physically viable. In the current paper, we reconsider the validity of such models by appealing to the so-called "causal entropy bound". In this framework, a paradoxical outcome is obtained: these brane worlds are indeed holographically viable, provided that the bulk charge is not too small. We go on to argue that this new finding is likely the more reliable one.

Keywords: Extra Large Dimensions, Black Holes, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Progress in understanding gravity, whether at a classical, semi-classical or quantum level, often requires a well-defined notion of what constitutes physically realistic matter. With guidance from the observable universe, most physicists would agree on matter that has a positive energy density (or, at least, a limit on the degree of negativity) and that preserves causality (i.e., signals should not exceed the speed of light). Historically speaking, these notions were put on a rigorous footing by the well-known energy conditions of general relativity [1]. From a modern perspective, the most prominently called upon of these constraints is the *null energy condition*, which can be expressed for perfect-fluid matter as follows:

$$\rho + p \geq 0\,, \tag{1.1}$$

where ρ is the energy density and p is the pressure. Since causality, by itself, further implies that $|p| \leq |\rho|$[1], the null energy condition really suggests that $\rho \geq 0$. (The only possible exception is when $\rho = -p < 0$, which is the well-understood case of a purely anti-de Sitter spacetime.)

At the classical level, the null energy condition holds up relatively well and has, at times, been elevated to almost the status of a fundamental principle. However, classical violations are known to occur; especially in theories that incorporate non-minimally coupled scalar fields, such as inflationary cosmology. At the quantum level, the situation is even worse, as violations of the null condition due to quantum effects are certainly not an uncommon event. (For recent discussions and relevant citations, see [3, 4].)

In spite of its somewhat dubious status, the null energy condition continues to be utilized in many phases of theoretical gravity research.[2] The motivating factor for this

[1]There are, however, subtle examples in which causality can be maintained even if $|p| \leq |\rho|$ is violated. See the introduction of [2] for an interesting discussion.

[2]Sometimes the *weak energy condition* [1] is alternatively employed, which is, with the assumption of causality, essentially the same as the null energy condition.

persistence is that the violations which do occur seem to be relatively small, implying that some sort of bound on negative energy densities must still be in effect. Unfortunately, what this bound should be, precisely, remains conspicuously unclear. (Averaging the energy densities over space and/or time is, however, one distinct possibility [5].)

If we accept that spacetimes with negative-energy matter can, in principle, exist (thereby rendering the null energy condition as obsolete), then a method of discriminating between the plausible and the implausible becomes a critically important issue. One possible recourse is to appeal to the holographic principle, which is believed to be a fundamental element of any viable theory of quantum gravity [6]–[8]. In essence, the holographic principle provides a bound on the amount of information or entropy that can be stored in a given region of spacetime. It thus follows that a clear violation of a suitably defined holographic bound — or some other manifestation of this paradigm, such as a holographically induced c-theorem [9] — should provide ample evidence that a given spacetime is physically unacceptable.[3]

There is a caveat that should be considered before one attempts to apply the holographic principle in the above manner. Namely, many formulations of this principle have used the null (or weak) energy condition as an antecedent. For instance, let us consider the *covariant holographic bound* as proposed by Bousso [10]. Although this bound is believed to have universal validity [8], it has only been rigorously established under certain conditions [11]; one of these being the null energy condition. That is to say, the covariant bound might require modifications (albeit, likely minor) if it is to be applied to spacetimes with a limited degree of negative-energy matter. The null (or weak) energy condition also figures prominently in the formulation of holographically induced c-theorems [9].

Notable examples of the "holographic discrimination" of *exotic* spacetimes[4] include recent works by Brustein *et al* [12] and McInnes [4]. What is of particular interest is that both of these treatments have employed aspects of holography which nicely circumvent the caveat discussed above. More specifically, the former utilized the *causal entropy bound* of Brustein and Veneziano [13], while the latter applied a holographic consistency condition that was first proposed by Bousso and Randall [14].

In the current paper, our primary objective is a similar type of holographic viability test on a certain class of exotic cosmologies; in particular, we will follow [12] in utilizing the causal entropy bound [13]. As for the model to be tested, it is essentially a Friedmann-Robertson-Walker (FRW) universe with radiative matter and dark energy, but also containing *stiff* matter with a negative energy density (i.e., matter for which $p = \rho < 0$). Such cosmologies are of interest for a couple of reasons. Firstly, they naturally arise in Randall-Sundrum-like brane-world scenarios [15] for which a three-brane (i.e., "our universe") is moving in the five-dimensional background of an anti-de Sitter Reissner-Nordstrom black hole.[5] In this regard, it is useful to keep in mind that, from the perspective of a brane

[3]Holographic bounds, however, can take on substantially different forms depending on the context. Hence, the applicability of holography in a given situation is not always straightforward. We discuss this point further in section 2.

[4]In this paper, exotic will often be used as a synonym for the presence of negative-energy matter.

[5]Some other treatments of brane cosmology with a bulk charge can be found in [16]–[25].

observer, the motion of the brane through the (otherwise) static bulk will appear as either a cosmological contraction or expansion [26]. Secondly, it has been shown that the effective brane cosmology is that of a "bouncing" FRW universe, completely devoid of singularities [24, 25]. Such bounce universes provide an intriguing means of circumventing the theoretical complications of the "big bang" (or "crunch").

To further motivate the upcoming analysis, let us take note of a recent paper (by the current author) [25]. In this work, we subjected the very same brane-world-induced bounce cosmologies to a holographic litmus test of a different kind. The basis for this test (also see [2, 27]) was a de Sitter-holographic c-theorem [28]–[31], which can be viewed as a consequence of the conjectured duality between de Sitter spacetimes and conformal field theories [32]. (It is relevant that the brane worlds in question are assumed to have a positive vacuum energy and, hence, are typically asymptotically de Sitter spacetimes.) Interestingly, the results of [25] imply that these cosmologies are *not* physically viable from a holographic perspective. However, because of the caveat discussed above, as well as some interpretative difficulties (elaborated on in section 5), there is significant doubt as to the reliability of the c-theorem in this particular context.

The remainder of the paper is organized as follows. In the next section, we discuss the causal entropy bound [13]; including its historical motivation, the underlying premise and the relevant formalism. In section 3, we focus on the brane-world scenario of interest; specifically, the pertinent qualitative and quantitative features of the induced FRW cosmologies. However, the formal derivations are left to the earlier works [24, 25]. In section 4, we rigorously examine the holographic feasibility of these spacetimes by way of the causal entropy bound. Here, it is shown that these bounce cosmologies are indeed viable, provided that the charge (on the bulk black hole) satisfies a lower bound. Finally, in section 5, we provide a brief summary and discussion; with particular emphasis on interpreting the current findings in view of the paradoxical implications of our prior results [25].

2. The causal entropy bound

The causal entropy bound [13] will be central to the subsequent analysis, so let us begin with a brief review of its motivation and then present the relevant formalism.

The holographic principle [6, 7] suggests that, for a closed system, there will be some finite upper bound on the entropy, S. On the basis of preserving the *generalized* second law of thermodynamics [33], Bekenstein proposed the following bound for a system of limited self-gravity:[6]

$$S < ER\,, \tag{2.1}$$

where E is the total enclosed energy and R is a characteristic length scale (for instance, the radius in the case of a spherical system). The above can be rewritten as

$$S < \frac{RR_S}{G_4}\,, \tag{2.2}$$

[6]For the time being, we restrict considerations to a four-dimensional spacetime and set all fundamental constants, except for Newton's constant (G_4), equal to unity. In this section, numerical factors (of the order unity) will often be ignored.

where $R_s \sim G_4 E$ is the Schwarzschild radius of the system. Moreover, because of the condition of limited gravity, $R > R_S$ and the Bekenstein bound directly implies

$$S < \frac{R^2}{G_4} \sim \frac{A}{G_4}\,, \tag{2.3}$$

where $A \sim R^2$ is the surface area. This is just the "usual" form of the holographic bound, which follows from the viewpoint that the entropy within a given volume should be maximized by a black hole of the same size [6, 7].

In historical retrospect, it was not at all clear as to how the above bounds should be extended to scenarios of strong or potentially strong gravity; in particular, cosmological situations. Prior to the advent of holography, Bekenstein [34] proposed that, for cosmological purposes, R in eq. (2.1) should be chosen as the particle horizon. This proposal, however, does lead to contradictions, and so Fischler and Susskind [35] suggested that eq. (2.3) should be modified so that the area of the particle horizon bounds the entropy contained in the past light cone. Although an improvement, the Fischler-Susskind bound still runs into problems, such as during the collapsing phase of a closed universe. Eventually, Bousso [10] generalized their proposal by considering the entropy bounded by light cones of diminishing cross-sectional area (which can be future and/or past directed depending on the system in question). The Bousso bound has the desirable features of being covariantly defined and having general applicability (i.e., not just cosmological scenarios), and it has yet to be contradicted in any physically realistic situation [8]. Nonetheless, the Bousso bound, which is formulated in terms of null surfaces, can *not* be universally applied to spacelike regions in any given spacetime.

With regard to the issue of holography in a spacelike cosmological region, an interesting proposal is the so-called *Hubble entropy bound* [36] (also see [37]). This bound is based on a pair of common-sense inputs: (i) the entropy in a given region of space is maximized by the largest black hole which can fit inside and (ii) in a cosmological background, the largest possible *stable* black hole has a radial size that roughly corresponds to the Hubble horizon (H^{-1}). By virtue of these observations, it immediately follows that, in a given spacelike region, the entropy should be bounded by the entropy of a Hubble-sized black hole *times* the number of Hubble-sized spheres that can fit inside of the total volume (V). That is:

$$S < \frac{H^{-2}}{G_4} \times \frac{V}{H^{-3}} = \frac{VH}{G_4}\,. \tag{2.4}$$

This Hubble entropy bound served as the primary motivation for the causal entropy bound of Brustein and Veneziano [13]. In fact, the causal bound can be viewed as a covariant generalization of the Hubble bound. More specifically, the Hubble horizon is replaced by a "causal connection scale", R_{CC}, that can be interpreted as the length scale above which spacetime perturbations are causally disconnected and, therefore, black holes can not (presumably) form. The determination of this length scale is a highly technical process, and we refer the readers to the seminal work [13] for the details and pertinent citations. Let us, however, quote the result in a form that is readily applicable to a standard (four-dimensional) FRW spacetime (for the covariantly defined equivalent, again see [13]):

$$R_{CC}^{-2} = \text{Max}\left[\dot{H} + 2H^2 + \frac{k}{a^2}, \quad -\dot{H} + \frac{k}{a^2}\right]. \tag{2.5}$$

Here, a dot denotes differentiation with respect to cosmological time (t), $a = a(t)$ is the FRW scale factor, $H = \dot{a}/a$ is the Hubble "constant", and k is the usual spatial-curvature parameter. Note that k takes on a value of 0, +1 or -1 for a flat, closed or open universe (respectively).

Actually, by assuming a perfect-fluid form for the spacetime matter and applying the well-known relations for the energy density (ρ) and pressure (p) [38],

$$H^2 = \frac{8\pi G_4}{3}\rho - \frac{k}{a^2}, \tag{2.6}$$

$$\dot{H} = -4\pi G_4 (\rho + p) + \frac{k}{a^2}, \tag{2.7}$$

one can translate eq. (2.5) into a very convenient form:

$$R_{CC}^{-2} = 4\pi G_4 \text{Max}\left[\frac{\rho}{3} - p, \quad \rho + p\right]. \tag{2.8}$$

It may be surprising that this particular expression has no explicit dependence on the curvature parameter, k.

Let us now be more explicit with regard to the proposed bound. The total entropy contained in a spacelike hypersurface of volume V should be bounded according to (cf. eq. (2.4))

$$S < \beta \frac{V R_{CC}^{-1}}{G_4}, \tag{2.9}$$

where β is a numerical factor of the order unity (reflecting the factors left out of eq. (2.4) and the inherent ambiguity in bounds of this nature). Note that the causal connection scale can, for the problems of interest, be calculated by way of eq. (2.5) or (2.8).

Although the causal entropy bound has a conjectural status, let us take note of the following supporting evidence: (i) violations of the bound require either trans-Planckian temperatures, matter sources with a negative energy density or an acausal equation of state [12] (all of which are arguably, but not necessarily, unphysical conditions), (ii) the bound closely follows Bousso's covariant bound [10] in situations where they can be compared [13], (iii) in fact, in its explicitly covariant form [13], the causal bound is closely related to a condition that was used by Flanagan *et al* [11] to derive the Bousso bound, (iv) the causal bound is parametrically equivalent to the various holographic bounds proposed by Verlinde [39] for a closed, radiation-dominated universe [40], (v) for conventional cosmologies, the bound essentially translates into $S < \sqrt{EV/G_4}$, a form that has also turned up in studies on uncertainty relations [41] and extensive thermodynamic systems [42].

It is useful to keep in mind that, although the causal bound works most favorably for positive-energy matter, it can still persist when some of the matter sources have a negative energy density [12]. That is to say, the causal bound can, in principle, be used to test the validity of exotic cosmological models that might otherwise be rejected on the basis of the null energy condition. It is this particular feature of the causal bound that makes it an ideal testing ground for the brane cosmologies discussed below.

JHEP05(2003)008

3. Induced brane-world cosmologies

Let us now formally introduce the model of interest, which can be viewed as a generalization of the Randall-Sundrum brane-world scenario [15]. More specifically, we will consider a 3+1-dimensional (positively curved[7]) brane moving in a 4+1-dimensional bulk spacetime that is otherwise *static*. Without loss of generality, the bulk geometry can be described by an anti-de Sitter black hole with an electrostatic charge and a constant-curvature horizon [17]. Such black holes are "Reisnner-Nordstrom-like", but with an arbitrary horizon topology [43].

In a general sense, the above type of picture is known to lead to a brane dynamical equation that mimics a FRW universe [26]. Moreover, the details of the bulk solution play a prominent role in determining what kinds of matter will be *holographically induced* on the brane world. (There should, of course, also be matter that lives strictly on the brane; most prominently, the contributions from the standard model. This matter can, however, be ignored for the current, "philosophical" discussion.) For a charged black hole bulk in particular, the brane world turns out to be a "bounce" cosmology [24, 25]. That is, the universe is asymptotically de Sitter in the far past, contracts to a non-vanishing minimum at some given time (which we take as being $t = 0$ without loss of generality) and then expands to an asymptotically de Sitter future.[8] The important point to keep in mind is that the bounce is strictly non-singular (i.e., the FRW scale factor does *not* vanish) provided that the bulk black hole has a non-vanishing charge, no matter how small.

A detailed discussion of this cosmological framework (at least for the asymptotic regimes) can be found in [24, 25] (also see [44, 45]), and we refer the interested reader to these citations. Here, we will only quote the formalism that is necessary for current purposes.

Firstly, let us take note of the bulk solution; namely, a five-dimensional anti-de Sitter Reissner-Nordstrom-like black hole:

$$ds_5^2 = -f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{k,3}^2 \,, \tag{3.1}$$

where

$$f(r) = \frac{r^2}{L^2} + k - \frac{\omega M}{r^2} + \frac{3\omega^2 Q^2}{16 r^4} \tag{3.2}$$

and

$$\omega \equiv \frac{16\pi G_5}{3\mathcal{V}_3} \,. \tag{3.3}$$

In the above, L is the curvature radius of the anti-de Sitter bulk, G_5 is the five-dimensional Newton constant (typically, $G_5 \sim LG_4$), and $\mathcal{V}_3$ is the dimensionless volume element associated with the three-dimensional (spacelike) constant-curvature hypersurface $d\Omega_{k,3}^2$. There

[7]This condition of positive curvature is for the sake of compliance with the empirical evidence [38] but is actually inconsequential to the arguments which follow.

[8]Actually, for the Reissner-Nordstrom ($k = +1$) case, a sufficiently massive bulk black hole may end up as an eternally oscillating universe rather than "escaping" to an asymptotically de Sitter cosmology at temporal infinity. This distinction has, however, no relevance to our subsequent analysis.

are also three constants of integration for this solution: (i) the discrete parameter k, which describes the horizon topology such that +1, 0 and -1 respectively correspond to a spherical, flat and hyperbolic geometry, (ii) the conserved mass of the black hole, M, which can be regarded as a strictly non-negative quantity[9] and (iii) the electrostatic charge, Q, of the black hole. For the duration, we will rather work with the dimensionless measure of charge

$$\epsilon^2 \equiv \frac{3Q^2}{4M^2}\,. \tag{3.4}$$

It can be shown that the existence of a pair of positive and real black hole horizons necessitates that $\epsilon^2 < 1$. Moreover, on intuitive grounds, we will assume that ϵ is at least an order of magnitude smaller than one.

Let us next focus our attention on the brane world. After some suitable identifications, one finds that the induced metric on the brane takes on the following FRW form:

$$ds_4^2 = -dt^2 + a^2(t)d\Omega_{k,3}^2\,, \tag{3.5}$$

where t measures the physical time from the point of view of a brane observer and $a = r(t)$ is the cosmological scale factor. The corresponding Friedmann equation was found to be as follows [25]:

$$H^2 = \frac{\Lambda_4}{3} - \frac{k}{a^2} + \frac{\omega M}{a^4} - \frac{\omega^2 M^2 \epsilon^2}{4a^6}\,, \tag{3.6}$$

where the Hubble constant, H, is as defined in the prior section, and Λ_4 is a constant that depends on both L and the tension of the brane. Obviously, Λ_4 plays the role of the (effective) cosmological constant in the brane universe. Lacking a microscopic theory, the best we can do is to fix Λ_4 according to the observational data [38]; hence, Λ will be regarded as being a positive quantity with an extremely small magnitude ($\sim 10^{-120}$ in Planck units).

Remarkably, eq. (3.6) is simply the four-dimensional Friedmann equation for radiative matter (the a^{-4} term) along with an exotic (negative-energy) stiff-matter source, the a^{-6} term. Note that this brane universe can be open, closed or flat ($k = -1, +1$ or 0) depending on the horizon topology of the bulk solution.

To help clarify matters, we can suggestively rewrite the Friedmann equation in the following manner:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_4}{3}\left[\rho_v + \rho_r + \rho_e\right], \tag{3.7}$$

where the *vacuum*, *radiation* and *exotic* energy densities have respectively been defined as follows:

$$\rho_v = \frac{\Lambda_4}{8\pi G_4}\,, \tag{3.8}$$

$$\rho_r = \frac{3M}{8\pi G_4}a^{-4}\,, \tag{3.9}$$

$$\rho_e = -\frac{3\omega^2 M^2 \epsilon^2}{32\pi G_4}a^{-6}\,. \tag{3.10}$$

[9]Technically speaking, the hyperbolic ($k = -1$) black hole solution supports a negative mass; however, this *very* exotic scenario will not be considered here.

JHEP05(2003)008

The corresponding pressures are also known by way of standard identifications [38]:

$$p_v = -\rho_v \,, \tag{3.11}$$
$$p_r = \frac{\rho_r}{3} \,, \tag{3.12}$$
$$p_e = \rho_e \,. \tag{3.13}$$

As discussed earlier, one finds [24, 25] that the brane cosmology describes a bounce universe: the scale factor reaches a finite minimum at $t = 0$ and is (typically) asymptotically de Sitter in both the distant past and far future. The bounce can be attributed to the negative-energy matter, which dominates at small values of a (thanks to the a^{-6} red-shift factor) and creates a significant enough repulsive force so that a "big crunch" is avoided.

Although the scale factor can not be explicitly solved for throughout its evolution (unless we set ϵ [44, 45] or Λ_4 [24] to vanish), the asymptotic regimes are well understood because of the discrepancy in the various red-shift factors. More to the point, the constant vacuum energy plays essentially no role near the bounce, whereas the other sources are rapidly diluted away as $|t|$ or a becomes large. For the current analysis, we are particularly interested in the solution near the bounce (see the following section), which is, up to negligible corrections, expressible as follows [24, 25]:

$$a^2 = \frac{\omega M}{2}\left(1 - \sqrt{1-\epsilon^2}\cos[2\eta]\right) \qquad \text{if} \quad k = +1 \,, \tag{3.14}$$
$$a^2 = \frac{\omega M}{4}\left(\epsilon^2 + 4\eta^2\right) \qquad \text{if} \quad k = 0 \,, \tag{3.15}$$
$$a^2 = \frac{\omega M}{2}\left(\sqrt{1+\epsilon^2}\cosh[2\eta] - 1\right) \qquad \text{if} \quad k = -1 \,, \tag{3.16}$$

where η is the usual conformal-time coordinate (i.e., $dt = ad\eta$).

It is clear from the above forms that, as long as the charge is non-vanishing (i.e., as long as $\epsilon^2 > 0$), a will never shrink to zero. At the bounce ($t = \eta = 0$) in particular, all three equations take the simple form

$$a^2(0) = \frac{\omega M \epsilon^2}{4} + \mathcal{O}[\epsilon^4] \,. \tag{3.17}$$

Substituting this outcome into eqs. (3.9) and (3.10), we are able to deduce that

$$\rho_r(0) = -\rho_e(0) = \frac{6}{\omega\pi G_4 M \epsilon^4} + \mathcal{O}\left[\epsilon^{-2}\right] , \tag{3.18}$$

which will prove to be useful later in the paper.

We would not be so bold as to suggest that the above framework represents a physically realistic cosmology. On the other hand, this scenario does have some desirable and interesting features. For instance, the four-dimensional cosmological constant can be regarded as an input from the bulk theory and, therefore, the *cosmological constant problem* [46] (i.e., how to explain the relative smallness of Λ_4) may have a natural resolution in terms of higher-dimensional dynamics. (This assumes that string or M theory can be used to predict the values of L and the brane tension and that this prediction complies with the

JHEP05(2003)008

empirical data. A very tall order indeed!) Furthermore, the feature of a bouncing universe would allow one to circumvent the issue of resolving the big bang (or crunch) singularity, which afflicts many (if not most) cosmological models.

On the other hand, the non-singular bounce depends on the existence of a negative-energy matter source, ρ_e, which may be an unappealing feature to some. It is, however, interesting to note that the *total* energy density does remain strictly non-negative[10] and the negative-energy source will quickly be diluted away by the cosmological expansion (by virtue of the a^{-6} red-shift factor). It is clear that, if one does not wish to call upon the rather dubious [3] null energy condition, some other means is necessary for testing the viability of such exotic cosmologies. In the next section, we will invoke the causal energy bound for just this purpose.

4. Causal entropy bound on the brane

As alluded to above, we now intend to employ the causal entropy bound, as discussed in section 2, as a means for assessing the feasibility of the exotic cosmologies in section 3. To apply the causal bound, one should choose a given spacelike hypersurface and then calculate both the upper bound (2.9) and the actual entropy contained on that surface. Ideally, every such surface in the spacetime should be tested; however, in our study, it is sufficient to consider the spacelike hypersurface at the bounce ($t = \eta = 0$). Significantly, this surface is precisely where the brane world will be most vulnerable to a violation of the bound.

An important issue to resolve is the means of calculating the actual entropy. For our model, there are two entropic contributions to consider: the radiative matter and the exotic stiff matter (the vacuum energy, of course, makes no contribution). For simplicity, we will only deal with the former source, as one might anticipate that radiation would make the dominant contribution to the entropy. Moreover, if we can verify that the bound is satisfied for the radiative entropy (S_r), the inclusion of any additional source only strengthens our finding.

To be more explicit, we want to determine if the inequality

$$\left(\frac{S_{CB}}{S_r}\right)^2 > 1 \tag{4.1}$$

can be satisfied at the bounce, where

$$S_{CB} \equiv \beta \frac{V R_{CC}^{-1}}{G_4} \tag{4.2}$$

has been defined in accordance with eq. (2.9). Let us also recall that the causal connection scale, R_{CC}, can be obtained from either eq. (2.5) or eq. (2.8), while V denotes the volume of the hypersurface in question.

[10] In spite of this feature of non-negativity, the null energy condition (1.1) is still clearly violated, as can be seen by way of eqs.(3.12), (3.13) and (3.18). Indeed, for an open or flat FRW universe, such a violation is a necessary pre-requisite for a non-singular bounce [48].

JHEP05(2003)008

As an initial step in this procedure, R_{CC} will be calculated by way of eq. (2.8). Including all three sources (vacuum, radiation and exotic) and applying eqs. (3.11)–(3.13) for the corresponding pressures, we find that

$$R_{CC}^{-2} = \frac{8\pi G_4}{3} \mathrm{Max}\left[2\rho_v - \rho_e, \quad 2\rho_r + 3\rho_e\right] . \tag{4.3}$$

Now focusing on the bounce surface in particular, we see from eq. (3.18) that the left-hand argument must be positive, whereas the right-hand argument is clearly negative. Hence:

$$R_{CC}^{-2} \approx \frac{8\pi G_4}{3} \rho_r , \tag{4.4}$$

where we have made explicit use of eq. (3.18), as well as the relative smallness of the vacuum contribution.

Let us next consider the entropy of the radiation, S_r. Here, it is helpful to recall the standard Stephan-Boltzmann thermodynamic relations for radiative matter (in thermal equilibrium at temperature T):

$$s_r \equiv \frac{S_r}{V} = \mathcal{N}\alpha_1 T^3 , \tag{4.5}$$

$$\rho_r = \mathcal{N}\alpha_2 T^4 , \tag{4.6}$$

where the α's are numerical factors of the order unity and $\mathcal{N}$ is the (effective) number of particle species. Eliminating T from this pair of equations, we have

$$S_r = V\mathcal{N}^{1/4}\alpha\rho_r^{3/4} , \tag{4.7}$$

where α is another factor of the order unity. It is somewhat unclear, in this case of holographically induced radiation, as to what the actual value of $\mathcal{N}$ might be. However, it is difficult to imagine that $\mathcal{N}$ would be significantly larger than 10^4; therefore, we will simply absorb $\mathcal{N}^{1/4}$ into the numerical factor, α.

Substituting eqs. (4.2), (4.4), (4.7) into eq. (4.1) and simplifying, we now obtain the following inequality:

$$\frac{8\pi}{3}\frac{\beta^2}{\alpha^2}\frac{\rho_r^{-1/2}}{G_4} > 1 . \tag{4.8}$$

We can make this relation more transparent by substituting for $\rho_r = \rho(0)$ (cf. eq. (3.18)),

$$\frac{8\pi}{3}\frac{\beta^2}{\alpha^2}\sqrt{\frac{\omega\pi M}{6G_4}}\epsilon^2 > 1 , \tag{4.9}$$

and then dropping the inconsequential numerical factors to give

$$\epsilon^2 > \frac{1}{M_p}\frac{1}{\sqrt{\omega M}} , \tag{4.10}$$

where $M_p \sim G_4^{-1/2}$ is the Planck mass.

It is now clear that, for the casual entropy bound to be protected, the parameter $|\epsilon|$ — or the magnitude of the charge in units of black hole mass — must not become too small.

Although somewhat counter-intuitive, this outcome makes sense if one considers that, in the proximity of the bounce, the exotic energy density goes as ϵ^{-4} (cf. eq. (3.18)), rather than the naive expectation of ϵ^2 (cf. eq. (3.10)). That is to say, a sufficiently strong charge is needed to prevent the negative energy from becoming too large in magnitude (at the bounce).

The need for a lower bound on the charge is certainly clear, but can we get a better feel for the numbers involved? First, let us consider the mass, M, of the bulk black hole. From the brane-world perspective, M should not be so large that it endangers early-universe cosmological considerations, such as nucleosynthesis (although these kinds of constraints are highly model dependent [47]). On the other hand, from the bulk point of view, one would expect that the black hole mass is large enough to insure stability. As is well known, an anti-de Sitter Schwarzschild black hole requires a radius of at least L to prevent a very rapid decay via Hawking radiation [49]. The analagous statement here (by an inspection of eq. (3.2)) is essentially $\omega M > L^2$. Combining the bulk and brane viewpoints, we can anticipate that $\omega M \sim L^2$, and so (roughly speaking):

$$\epsilon^2 > \frac{L^{-1}}{M_p} . \tag{4.11}$$

What about L? The value of the bulk curvature in brane-world scenarios is a highly model-dependent feature [47]: L^{-1} is usually taken as being anywhere from the electroweak scale (the lower limit allowed by observation) up to the four-dimensional Planck scale itself. In all likelihood, the "true" value would be somewhere in between; however, for the sake of argument, let us consider the optimistic electroweak limit. In this case:

$$\epsilon^2 > 10^{-16} ; \tag{4.12}$$

meaning that the charge to mass ratio of the black hole can still be negligibly small without jeopardizing the causal entropy bound.

5. Conclusion

In summary, we have "re-opened the case" on an intriguing class of brane-world scenarios. To elaborate, we have been investigating the validity, from a holographic perspective, of a positively curved brane world evolving in the background of a Reissner-Nordstrom-like black hole. These induced cosmologies are particularly of interest for a couple of reasons. Firstly, as long as the bulk charge is non-vanishing, the brane world is completely free of singularities. That is, the brane universe "bounces" when a finite minimal size is reached and thereby avoids the unpleasant implications of a big bang or crunch. Secondly, the holographically induced matter includes an exotic, negative-energy contribution. This exotic stiff matter arises as a direct consequence of the electrostatic charge in the bulk black hole, and it is clear that the above two features (bouncing universe and negative-energy source) are, in fact, closely related.

In a previous paper [25], we utilized a de Sitter-generalized c-theorem [28]–[31] to argue that the same class of cosmologies is not holographically viable. (For closely related works, see [2, 27].) It is not, however, absolutely clear that the proposed c-theorem, which has the weak energy condition as an antecedent, can be directly applied to a spacetime that *a priori* violates this condition. It is also not apparent what a violation of this c-theorem is precisely supposed to signify. In this regard, it has been suggested by Strominger [28] that the associated renormalization-group flow is dually related to the flow of time in the cosmology. However, the ambiguous nature of time in quantum gravity [50] casts some doubt on the universal applicability of such an interpretation. Furthermore, the c-theorem in question can be viewed as a manifestation of the "dS/CFT correspondence" [32], and, perhaps significantly, this conjectured duality has been recently challenged on conceptual grounds [51].

In view of such interpretative difficulties, we felt compelled to re-examine the viability of these exotic cosmologies. Again calling on holography, we have applied the causal entropy bound [13]; specifically, at the most vulnerable point in the evolution of the brane world. (This being at the bounce, where the negative-energy matter plays its most dominant role.) From the perspective of this holographic bound, there appears to be no significant difficulties with the cosmologies of interest. It has also been established that the bulk charge must satisfy a lower bound; however, this bound was shown to be not particularly stringent for a wide range of physically interesting scenarios.

Unlike the priorly discussed c-theorem analysis, the causal entropy bound has no interpretative difficulties. Rather, this entropic bound follows from straightforward arguments of causality and holography at its most fundamental level. It is also significant that the causal entropy bound does not employ the null (or weak) energy condition as an explicit premise. It is this feature, in particular, that makes the causal bound an especially useful tool for discriminating spacetimes with negative-energy sources.

In spite of the above points, it is still quite possible that the c-theorem violation should be taken seriously enough to censor against these exotic cosmologies. Nonetheless, the current study casts significant doubt on this viewpoint and signifies that, if nothing else, the status of negative-energy matter (in a cosmological framework) should remain a very open question.

Acknowledgments

The author would like to thank V.P. Frolov for helpful conversations. The author also expresses his gratitude to M.E. Medved for her patience and encouragement.

References

[1] See, for instance, R.M. Wald, *General relativity*, University of Chicago Press, 1984.

[2] B. McInnes, *The dS/CFT correspondence and the big smash*, *J. High Energy Phys.* **08** (2002) 029 [hep-th/0112066].

[3] C. Barcelo and M. Visser, *Twilight for the energy conditions?*, *Int. J. Mod. Phys.* **D 11** (2002) 1553 [gr-qc/0205066].

[4] B. McInnes, *The covariant entropy bound, brane cosmology and the null energy condition*, *J. High Energy Phys.* **12** (2002) 053 [hep-th/0212014].

[5] L.H. Ford and T.A. Roman, *Averaged energy conditions and quantum inequalities*, *Phys. Rev.* **D 51** (1995) 4277 [gr-qc/9410043]; *The quantum interest conjecture*, *Phys. Rev.* **D 60** (1999) 104018 [gr-qc/9901074].

[6] G. 't Hooft, *Dimensional reduction in quantum gravity*, gr-qc/9310026.

[7] L. Susskind, *The world as a hologram*, *J. Math. Phys.* **36** (1995) 6377 [hep-th/9409089].

[8] R. Bousso, *The holographic principle*, *Rev. Mod. Phys.* **74** (2002) 825 [hep-th/0203101].

[9] See, for instance, O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111], and references therein.

[10] R. Bousso, *A covariant entropy conjecture*, *J. High Energy Phys.* **07** (1999) 004 [hep-th/9905177].

[11] E.E. Flanagan, D. Marolf and R.M. Wald, *Proof of classical versions of the Bousso entropy bound and of the generalized second law*, *Phys. Rev.* **D 62** (2000) 084035 [hep-th/9908070].

[12] R. Brustein, S. Foffa and A.E. Mayo, *Causal entropy bound for non-singular cosmologies*, *Phys. Rev.* **D 65** (2002) 024004 [hep-th/0108098].

[13] R. Brustein and G. Veneziano, *A causal entropy bound*, *Phys. Rev. Lett.* **84** (2000) 5695 [hep-th/9912055].

[14] R. Bousso and L. Randall, *Holographic domains of anti-de Sitter space*, *J. High Energy Phys.* **04** (2002) 057 [hep-th/0112080].

[15] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[16] C. Barcelo and M. Visser, *Living on the edge: cosmology on the boundary of anti-de Sitter space*, *Phys. Lett.* **B 482** (2000) 183 [hep-th/0004056].

[17] C. Csáki, J. Erlich and C. Grojean, *Gravitational Lorentz violations and adjustment of the cosmological constant in asymmetrically warped spacetimes*, *Nucl. Phys.* **B 604** (2001) 312 [hep-th/0012143].

[18] A.K. Biswas and S. Mukherji, *Holography and stiff-matter on the brane*, *J. High Energy Phys.* **03** (2001) 046 [hep-th/0102138].

[19] Y.S. Myung, *Standard cosmology from the brane cosmology with a localized matter*, *Mod. Phys. Lett.* **A 17** (2002) 1915 [hep-th/0103241];
R.-G. Cai, Y.S. Myung and N. Ohta, *Bekenstein bound, holography and brane cosmology in charged black hole background*, *Class. and Quant. Grav.* **18** (2001) 5429 [hep-th/0105070].

[20] D.H. Coule, *Is anti-de Sitter a realistic bulk in brane cosmology?*, gr-qc/0104016.

[21] D. Youm, *The cardy-verlinde formula and charged topological AdS black holes*, *Mod. Phys. Lett.* **A 16** (2001) 1327 [hep-th/0105249].

[22] M. Cvetič, S. Nojiri and S.D. Odintsov, *Black hole thermodynamics and negative entropy in desitter and anti-desitter Einstein-gauss-bonnet gravity*, *Nucl. Phys.* **B 628** (2002) 295 [hep-th/0112045];
J.E. Lidsey, S. Nojiri and S.D. Odintsov, *Braneworld cosmology in (anti)-de Sitter Einstein-Gauss-Bonnet-Maxwell gravity*, *J. High Energy Phys.* **06** (2002) 026 [hep-th/0202198].

[23] J.P. Gregory and A. Padilla, *Exact braneworld cosmology induced from bulk black holes*, *Class. and Quant. Grav.* **19** (2002) 4071 [hep-th/0204218].

[24] S. Mukherji and M. Peloso, *Bouncing and cyclic universes from brane models*, *Phys. Lett.* **B 547** (2002) 297 [hep-th/0205180].

[25] A.J.M. Medved, *Bad news on the brane*, hep-th/0205251.

[26] P. Kraus, *Dynamics of anti-de Sitter domain walls*, *J. High Energy Phys.* **12** (1999) 011 [hep-th/9910149].

[27] A.J.M. Medved, *How not to construct an asymptotically de Sitter universe*, *Class. and Quant. Grav.* **19** (2002) 4511 [hep-th/0203191].

[28] A. Strominger, *Inflation and the dS/CFT correspondence*, *J. High Energy Phys.* **11** (2001) 049 [hep-th/0110087].

[29] V. Balasubramanian, J. de Boer and D. Minic, *Mass, entropy and holography in asymptotically de Sitter spaces*, *Phys. Rev.* **D 65** (2002) 123508 [hep-th/0110108].

[30] F. Leblond, D. Marolf and R.C. Myers, *Tall tales from de Sitter space, I. Renormalization group flows*, *J. High Energy Phys.* **06** (2002) 052 [hep-th/0202094].

[31] K.R. Kristjansson and L. Thorlacius, *Cosmological models and renormalization group flow*, *J. High Energy Phys.* **05** (2002) 011 [hep-th/0204058].

[32] A. Strominger, *The dS/CFT correspondence*, *J. High Energy Phys.* **10** (2001) 034 [hep-th/0106113].

[33] J.D. Bekenstein, *A universal upper bound on the entropy to energy ratio for bounded systems*, *Phys. Rev.* **D 23** (1981) 287.

[34] J.D. Bekenstein, *Is the cosmological singularity thermodynamically possible?*, *Int. J. Theor. Phys.* **28** (1989) 967.

[35] W. Fischler and L. Susskind, *Holography and cosmology*, hep-th/9806039.

[36] G. Veneziano, *Pre-bangian origin of our entropy and time arrow*, *Phys. Lett.* **B 454** (1999) 22 [hep-th/9902126]; *Entropy bounds and string cosmology*, hep-th/9907012.

[37] R. Easther and D.A. Lowe, *Holography, cosmology and the second law of thermodynamics*, *Phys. Rev. Lett.* **82** (1999) 4967 [hep-th/9902088];
D. Bak and S.-J. Rey, *Cosmic holography*, *Class. and Quant. Grav.* **17** (2000) L83 [hep-th/9902173];
N. Kaloper and A.D. Linde, *Cosmology vs. holography*, *Phys. Rev.* **D 60** (1999) 103509 [hep-th/9904120].

[38] See, for instance, S.M. Carroll, *TASI lectures: cosmology for string theorists*, hep-th/0011110.

[39] E. Verlinde, *On the holographic principle in a radiation dominated universe*, hep-th/0008140.

JHEP05(2003)008

[40] R. Brustein, S. Foffa and G. Veneziano, *CFT, holography and causal entropy bound*, *Phys. Lett.* **B 507** (2001) 270 [hep-th/0101083].

[41] N. Sasakura, *An uncertainty relation of space-time*, *Prog. Theor. Phys.* **102** (1999) 169 [hep-th/9903146].

[42] G. Gour, *Extensive entropy bounds*, gr-qc/0212087.

[43] D. Birmingham, *Topological black holes in anti-de Sitter space*, *Class. and Quant. Grav.* **16** (1999) 1197 [hep-th/9808032].

[44] A.C. Petkou and G. Siopsis, *ds/CFT correspondence on a brane*, *J. High Energy Phys.* **02** (2002) 045 [hep-th/0111085].

[45] A.J.M. Medved, *Big bangs and bounces on the brane*, hep-th/0205037.

[46] See, for instance, S. Weinberg, *The cosmological constant problems*, *Rev. Mod. Phys.* **61** (1989) 1; *The cosmological constant problems*, talk given at *Dark matter 2000*, February, 2000 astro-ph/0005265.

[47] See, for instance V.A. Rubakov, *Large and infinite extra dimensions: an introduction*, *Phys. Usp.* **44** (2001) 871 [hep-ph/0104152].

[48] C. Molina-Paris and M. Visser, *Minimal conditions for the creation of a Friedman-Robertson-Walker universe from a 'bounce'*, *Phys. Lett.* **B 455** (1999) 90 [gr-qc/9810023].

[49] S.W. Hawking and D.N. Page, *Thermodynamics of black holes in anti-de Sitter space*, *Commun. Math. Phys.* **87** (1983) 577.

[50] See, for instance, C.J. Isham, *Canonical quantum gravity and the problem of time*, gr-qc/9210011.

[51] L. Dyson, J. Lindesay and L. Susskind, *Is there really a de Sitter/CFT duality*, *J. High Energy Phys.* **08** (2002) 045 [hep-th/0202163];
L. Susskind, *Twenty years of debate with stephen*, hep-th/0204027.